Part A

1. **Derive an expression for the spectrum Xc(j), Plot xc(t) and Xc(j Ω) for some values of a and**

CTFT for the signal:

If

Plots in Matlab

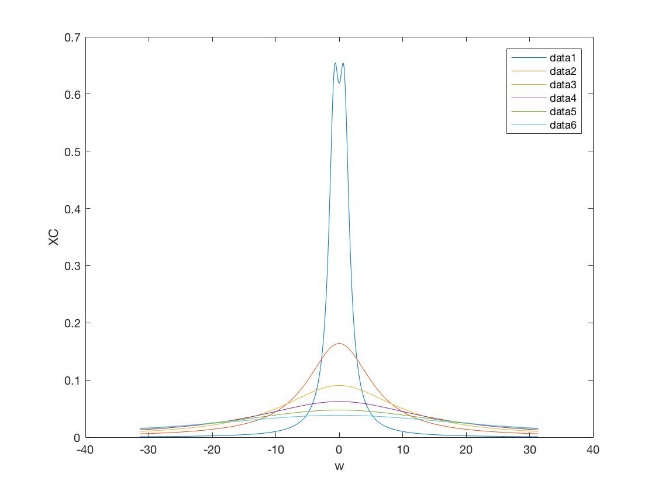
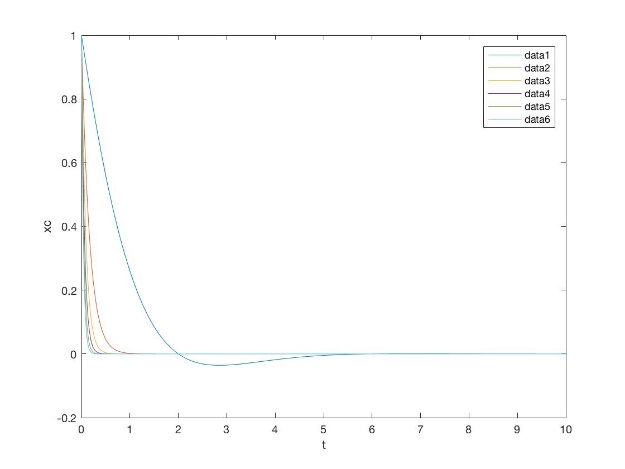
Fix , change . The diagram of and showed as below: 

Figure: signal in time domain Figure: signal in frequency domain

Fix , change. The diagram of and showed as below:

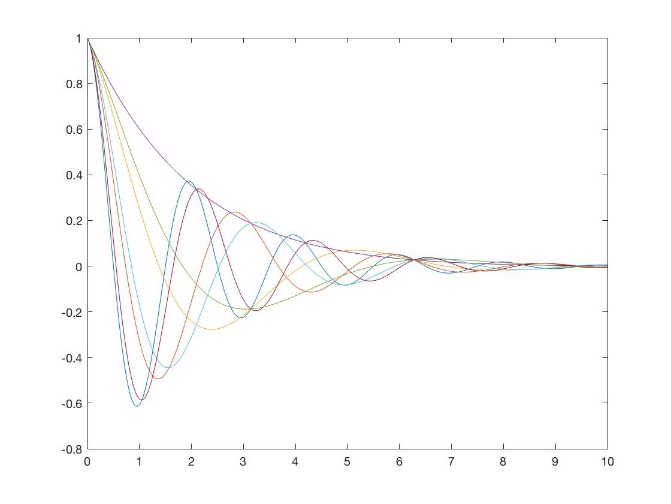
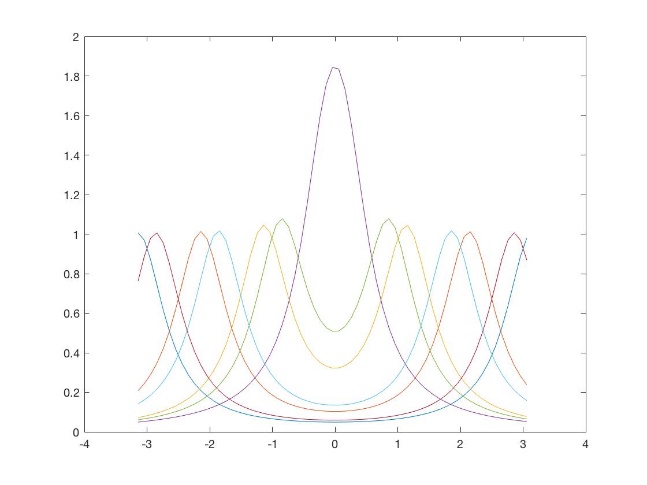
 

Figure: signal in time domain Figure: signal in frequency domain

From the diagram, it can be observed that if keep the value of , then changing , for , the rise time and set time are changed with , for , the peak value increases with , but the number of peak decreases as goes up. While if is changing, is a constant.

From the graphs, they show that there are more vibration before the curve coms to a stable value in . Meanwhile, with the changing of , the number of peak decreases as the value of close to 0, while the curve becomes lower with the increasing of .

The code in Matlab to get the diagram

Fix , change :

clc

clear all

%a = 0.12;

for a = (1:5:30);

    OMG = 0.25\*pi;

    %xc = exp(-a.\*t).\*cos(OMG1.\*t);

    t = (0:0.01:10);

    w=(-10\*pi:0.1:10\*pi);

    xc = exp(-a.\*t).\*cos(OMG.\*t);

    XC = (a+1i\*w)./((a+1i\*w).^2+ OMG.^2);

    figure(1);

    plot(t, xc);

    xlabel('t');

    ylabel('xc')

    hold on

    figure(2);

    plot(w, XC);

    xlabel('w');

    ylabel('XC')

    hold on

end

Fix , change:

clc

clear all

a = 0.5;

for OMG = -pi:pi

    %OMG = 0.25\*pi;

    %xc = exp(-a.\*t).\*cos(OMG1.\*t);

    t  = [0:0.01:10];

    xc = exp(-a.\*t).\*cos(OMG.\*t);

    figure(1);

    plot(t, xc);

    hold on

    figure(2);

    w = [-pi:0.1:pi];

    XC = (a+1i\*w)./((a+1i\*w).^2+ OMG.^2);

    plot(w, XC);

    hold on

end

b)

If

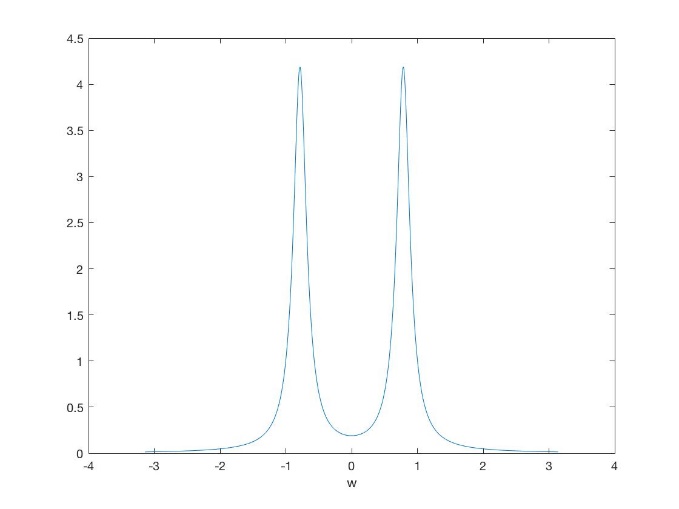
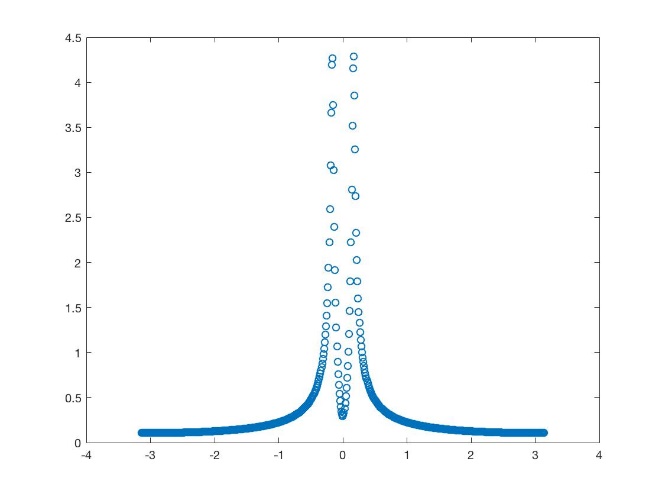
DTFT of is

If ,

DTFT of is

c)

The diagram of The diagram of

The command **freqz ,** [f w] = freqz (A , B , w), returns the frequency response vector f, and the corresponding angular frequency vector w, for the digital filter with numerator and denominator polynomial coefficients stored in B and A, respectively.

The code in Matlab to plot

%XC after sampling is

clear all

clc

close all

a = 0.12;

OMG = 0.25\*pi;

T = 1/4.8;

%fundamental frequency range

w = -pi:0.01:pi;

XC = (a+1i\*w)./((a+1i\*w).^2+ OMG.^2);%continues time FT

%abs(xc) only plot the magnetude

%NOW plot the sampled DTFT

A = [1 -exp(-a\*T)\*cos(OMG\*T) 0];

B = [1 -2\*exp(-a\*T)\*cos(OMG\*T) exp(-2\*a\*T)];

[f w] = freqz (A , B , w);

hold on;

%w/T=continuous time frequency

%abs(f)\*T

figure(2);

plot ( w, abs(f)\*T, 'o');

hold on

end

**d) Stability of the filter**

Proof:

If

From Euler Formula

Focus on the stability

Because

The only pole of the signal is ;

So and ;

Plot and

Diagram of

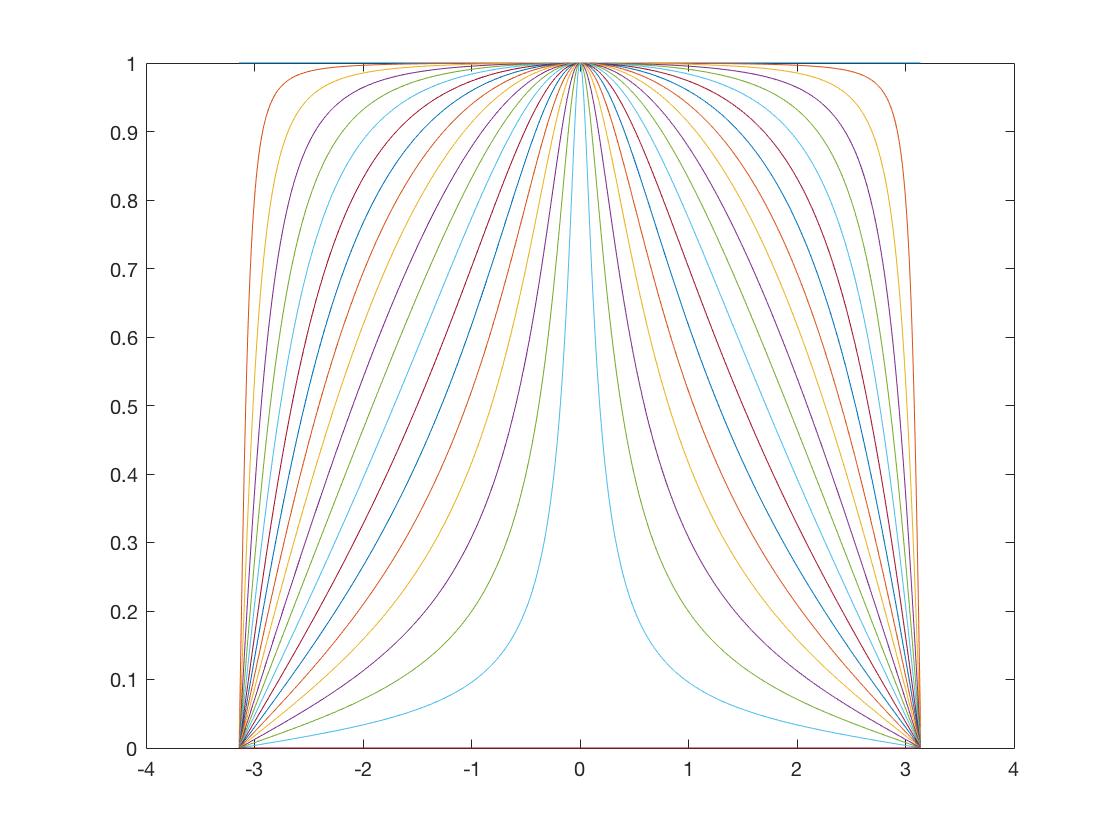
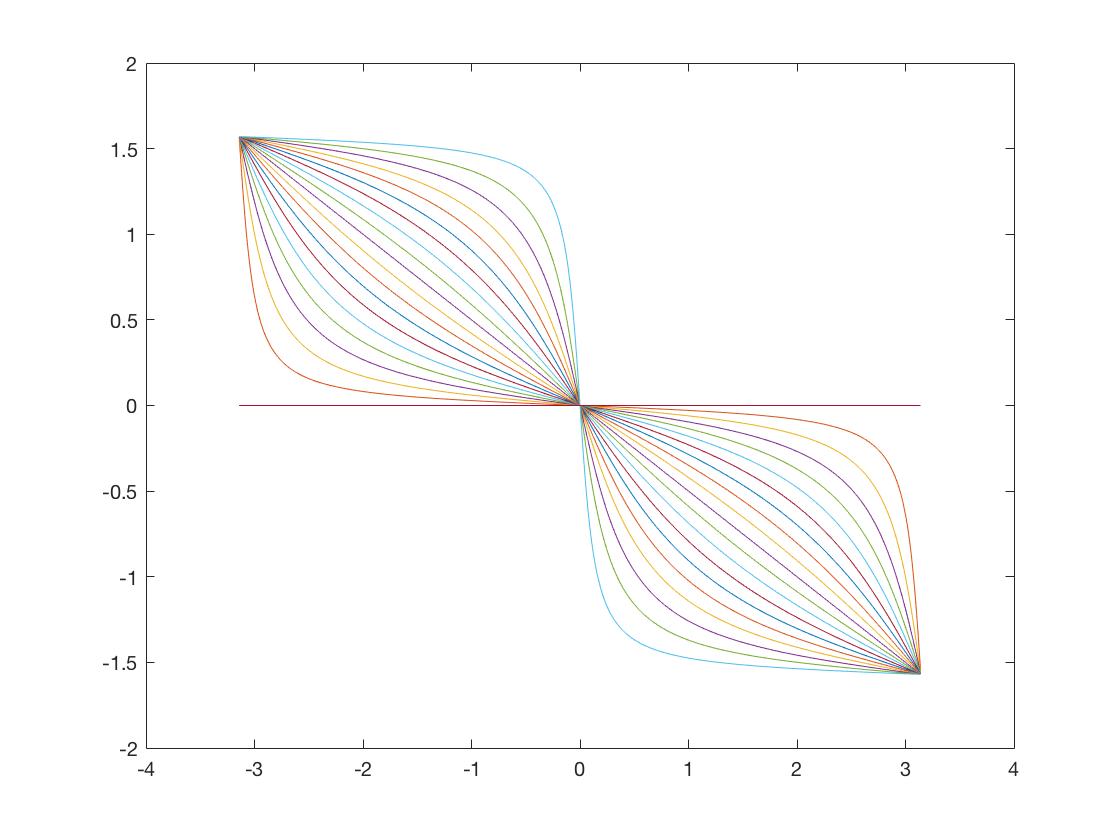


Diagram of



From the diagram above it can be observed that as the value of changing from -1 to 1, the peak of diagram of does not change, but the value of [-1,0] and [0,1] are closer to 0 as the increasing of .

When talking to diagram of , if the value of , then the curve of is just like the function y=-x. When , the relative curves of are all below the curve which has . As the becomes smaller, the curves are closer to real axis. When the , the relative curves are all above the curve which has . With the increasing of value of , these curves are closer to the imaginary axis.

The code in Matlab to plot the diagram of and

clear all

clc

for a = -1:0.1:1;

    H = zeros(0, 629);

    i = 1;

    for w = -pi :0.01: pi

        z = exp(j\*w);

        H(i) = (1 - a) / 2 \* (1 + 1/z) / (1 - a/z);

        i = i+1;

    end

     figure(1);

     plot(-pi :0.01: pi, abs(H));

     hold on;

    figure(2);

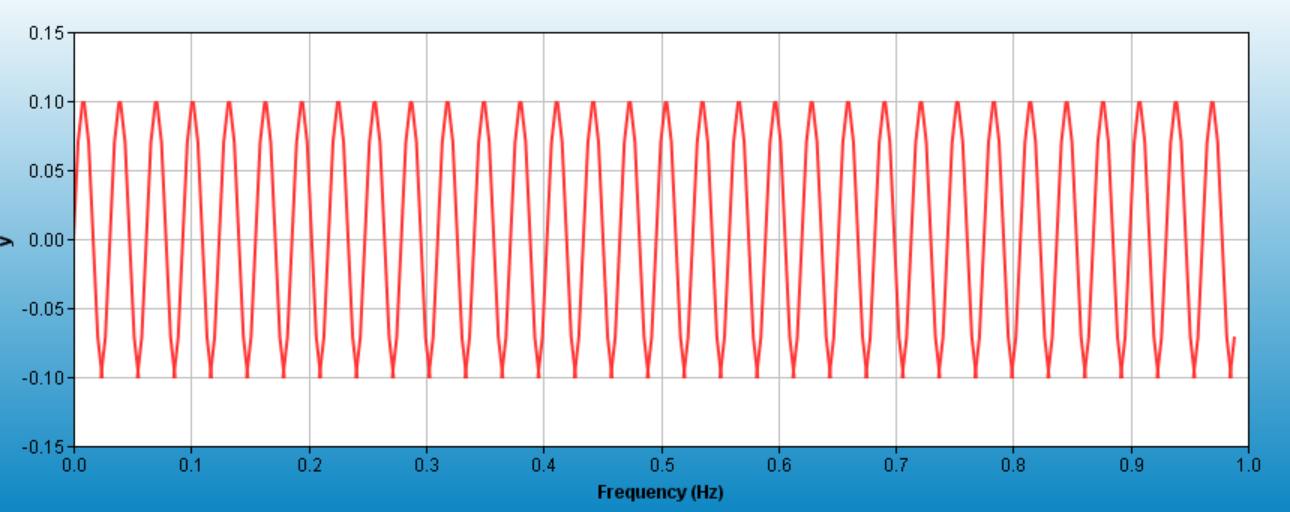
    plot(-pi :0.01: pi, angle(H));

    hold on;

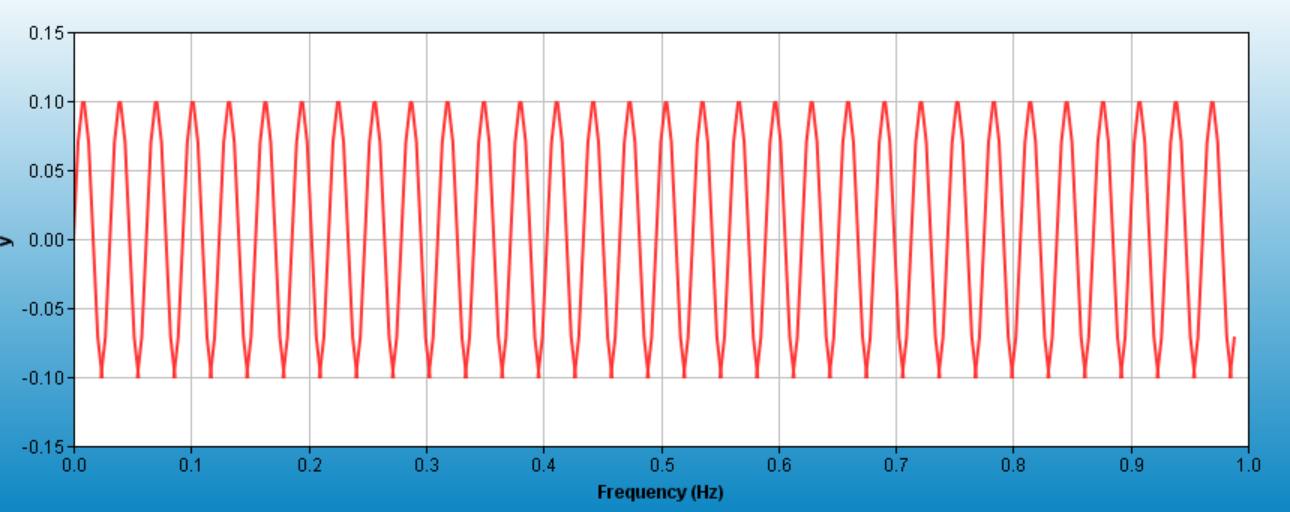
B. Implementation of digital anti-aliasing filters on a DSP

1. **Plot of the sample signal x(n) = 0.1 \* sin(0.25 \* 1.0 \* n);**

The Time domain representation of the signal has been ploteed in CCES as the figure below, where its x axis is time(s) and y axis is amplitude.



The frequency domain representation of the signal is shown in the figure below, where the x axis is frequency(Hz), y axis is amplitude.



1. **Write a c-program which generates two sequences of 256 samples of x(t), using the sampling frequencies F1 = 1.2Hz and F2 = 4.8Hz.**

**The code used to generate the 2 signals are attached below.**

#include <stdio.h>

#include <math.h>

*// Globals*

#define N 256

#define PI 3.1415

float x1[N]; *// Sample at T1*

float x2[N]; *// Sample at T2*

int main(void)

{

int i;

float omega1 = 0.25 \* PI, omega2 = 1.9 \* PI;

float T2 = 1/4.8;

float T1 = 1/1.2;

float a=0.12;

float alpha1 = 0.593, alpha2 = 0.464;

x1[0] = exp(-a\*0\*T1)\*cos(omega1\*0\*T1) + 0.1\*sin(omega2\*0\*T1);

x2[0] = exp(-a\*0\*T2)\*cos(omega1\*0\*T2) + 0.1\*sin(omega2\*0\*T2);

**for** (i = 0; i < N; i++)

{

x1[i] = exp(-a\*i\*T1)\*cos(omega1\*i\*T1) + 0.1\*sin(omega2\*i\*T1);

x2[i] = exp(-a\*i\*T2)\*cos(omega1\*i\*T2) + 0.1\*sin(omega2\*i\*T2);

}

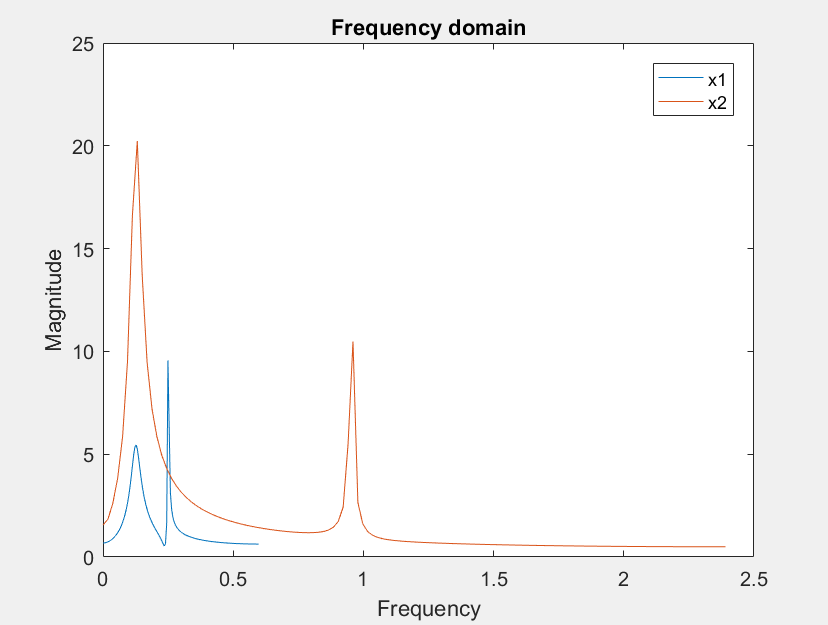
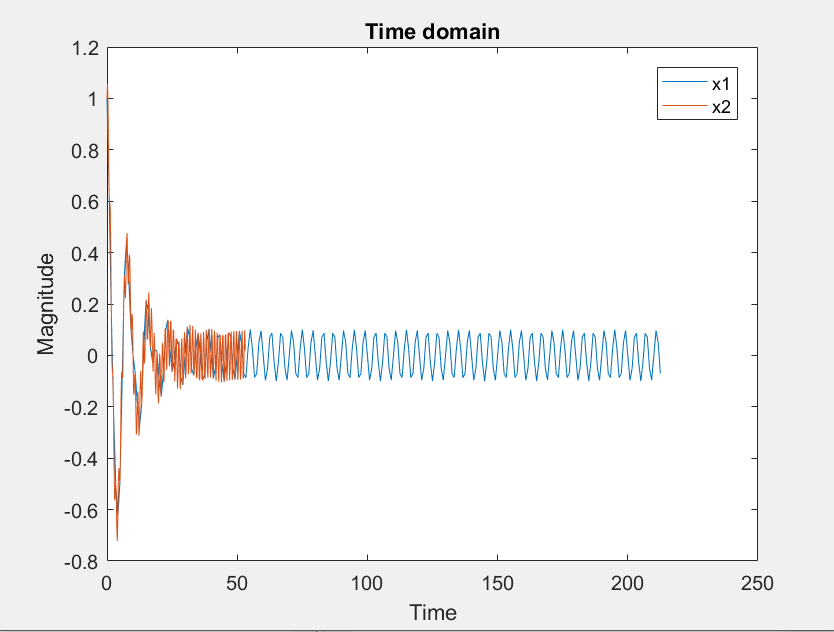
printf("Done.**\n**");

**return** 0;

}

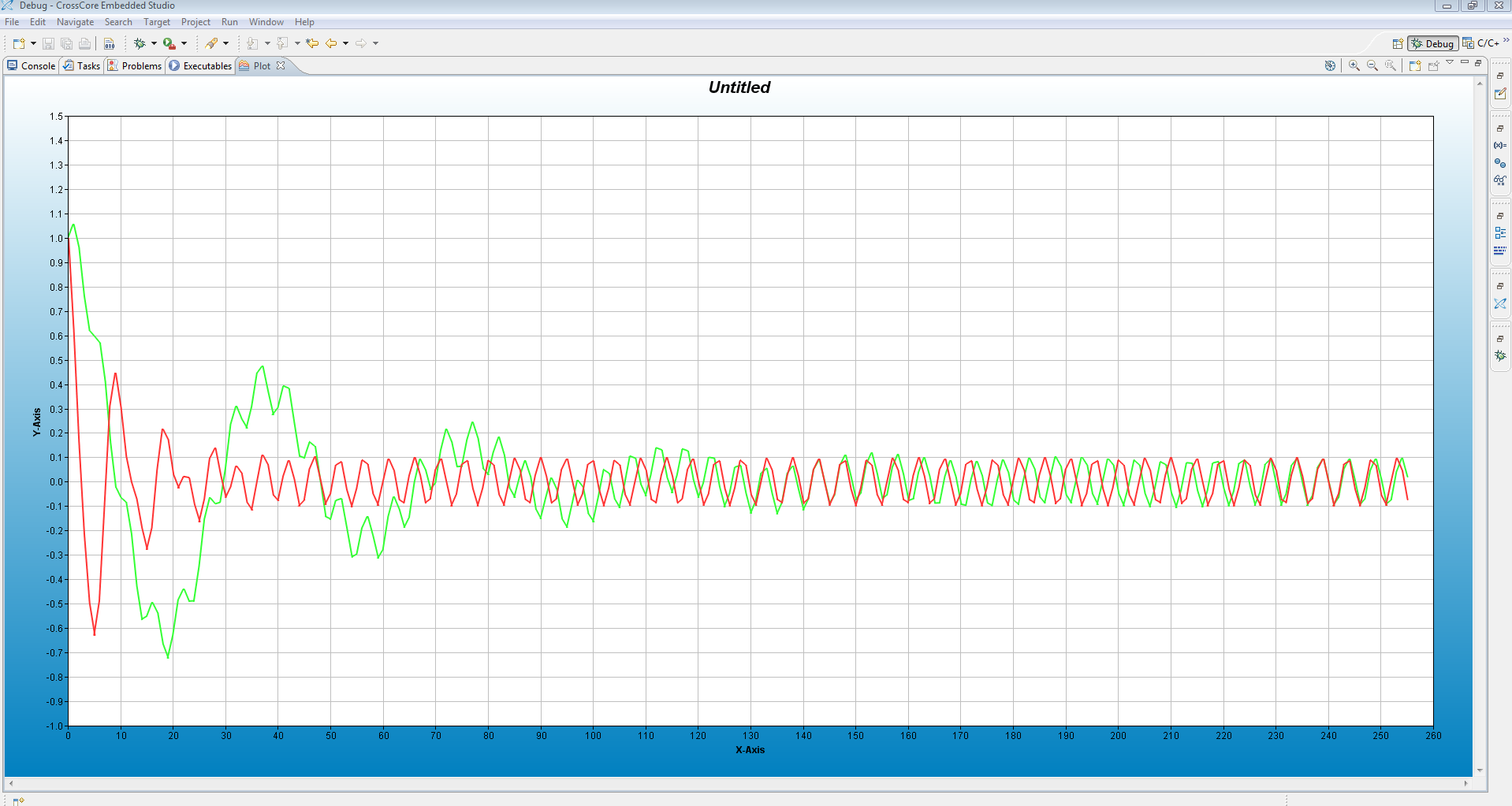
1. **Use the plot facility within CCES to plot the sampled signals in the time domain and in the frequency domain. Comment on the results.**

To predict how the signal is going to behave, Matlab was used to plot x1 and x2 in time and frequency domain. (Code attached in appendix).

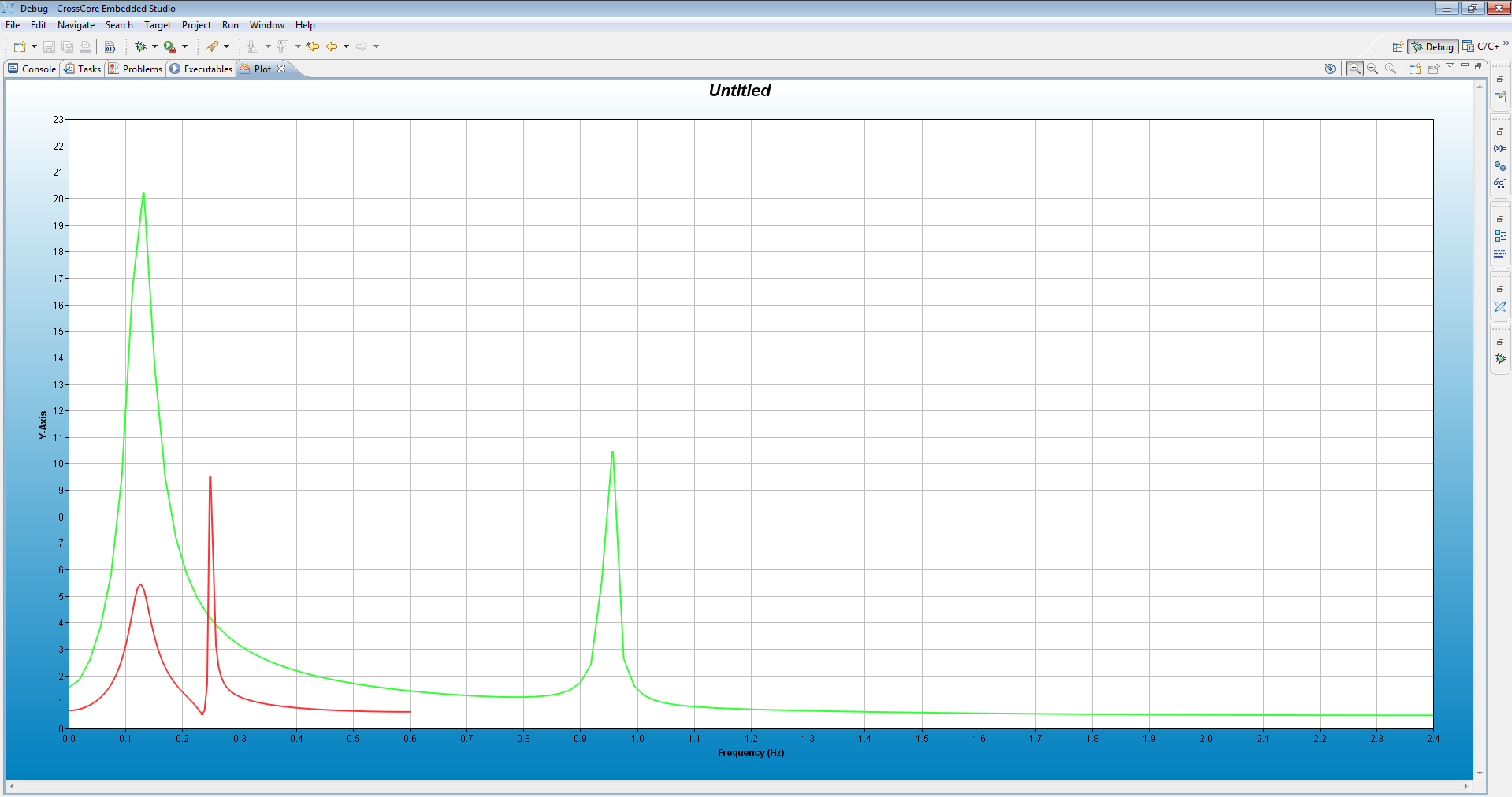


**The result generated by CESS are shown as below**

**Time domain plot. x axis: sample time, y axis: magnitude(linear scale)**



**Frequency domain plot. x axis: frequency(Hz), y axis: magnitude(linear scale)**



**As the plots above show, the signal generated in the DSP board are identical to that simulated in Matlab.**

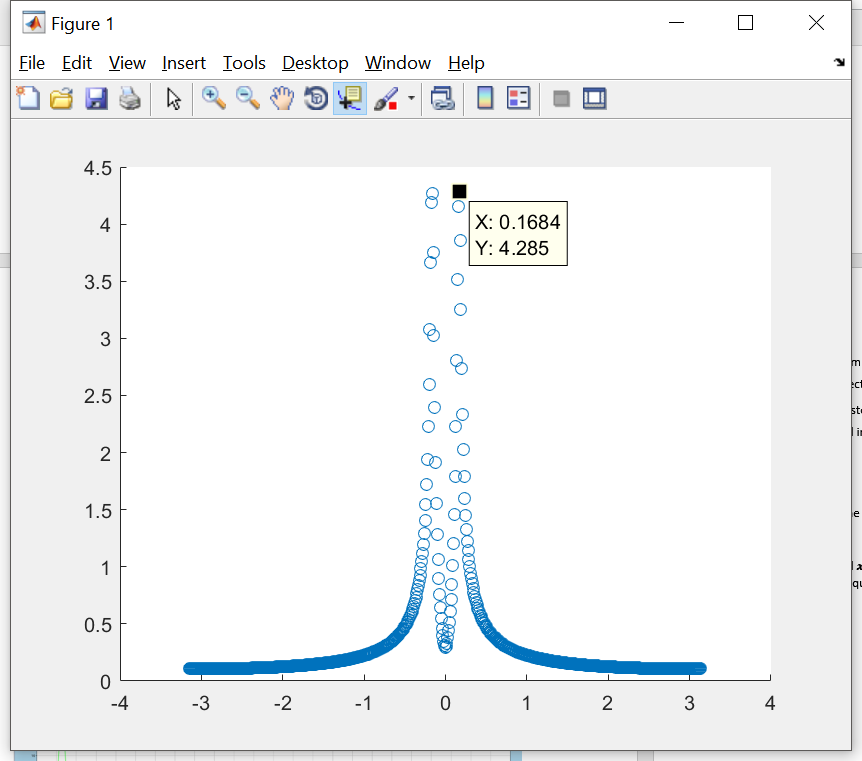
**Comment on the result:**

Theoretically speaking, the signal should generate two peaks at , and . Which is matching up with what signal x2(the green trace) in the CCES FFT plot.

However, there are two facts about the signal x1’s FFT plot were noticed in the FFT plot: 1. The 0.95Hz peak was missing, 2. There is an “unpredicted” peak between the region of 0.2-0.3Hz. Fact 1 might due to the sampling angular velocity . When the sampling frequency is less than of the signal, high frequency component of the signal will be distorted while sampling. Fact 2 is due to the fact that the sampling frequency is too low and it cause the folding effect around The frequency component from is distorted and folded back to 0.6-(0.95-0.6) = 0.25Hz, which fell into the 0.2-0.3Hz region we mentioned in the last paragraph.

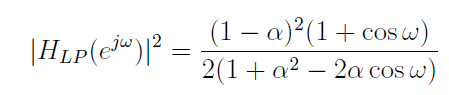
1. **Analysis of the first order and second order filter**

After DTFT, the signal , has been plotted in frequency domain in question part A(c), and can be obtained from the plot.



1. Find of the first order filter

It has been proven that



The gain(magnitude)=0.95 of this filter at ,

can be calculated by

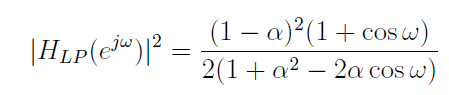
Solving this equation at Wolfram Alpha gives us

As the filter is designed to filter out 5% of the gain of x1, now we need to evaluate the gain of the signal after the filter.

Recall in discrete frequency domain, there will be a peak at rad/s.

As the requirement of the filter is to get a gain less than 0.25 at , the calculation above has shown that this filter won’t satisfy the design requirement.

1. Find of the second order filter



Since ,

The gain(magnitude)=0.95 of this filter at ,

can be calculated by

Solving this equation at Wolfram Alpha gives us .

As the filter is designed to filter out 5% of the gain of x1, now we need to evaluate the gain of the signal after the filter.

Gain = 0.207 is less than 0.25 therefore this filter will satisfy the design specification.

1. **Implementation of the two filters in C**

**Filter时域的公式推导过程麻烦耶写一下哈**

For the signal x[n], x[n]=x(nt)==

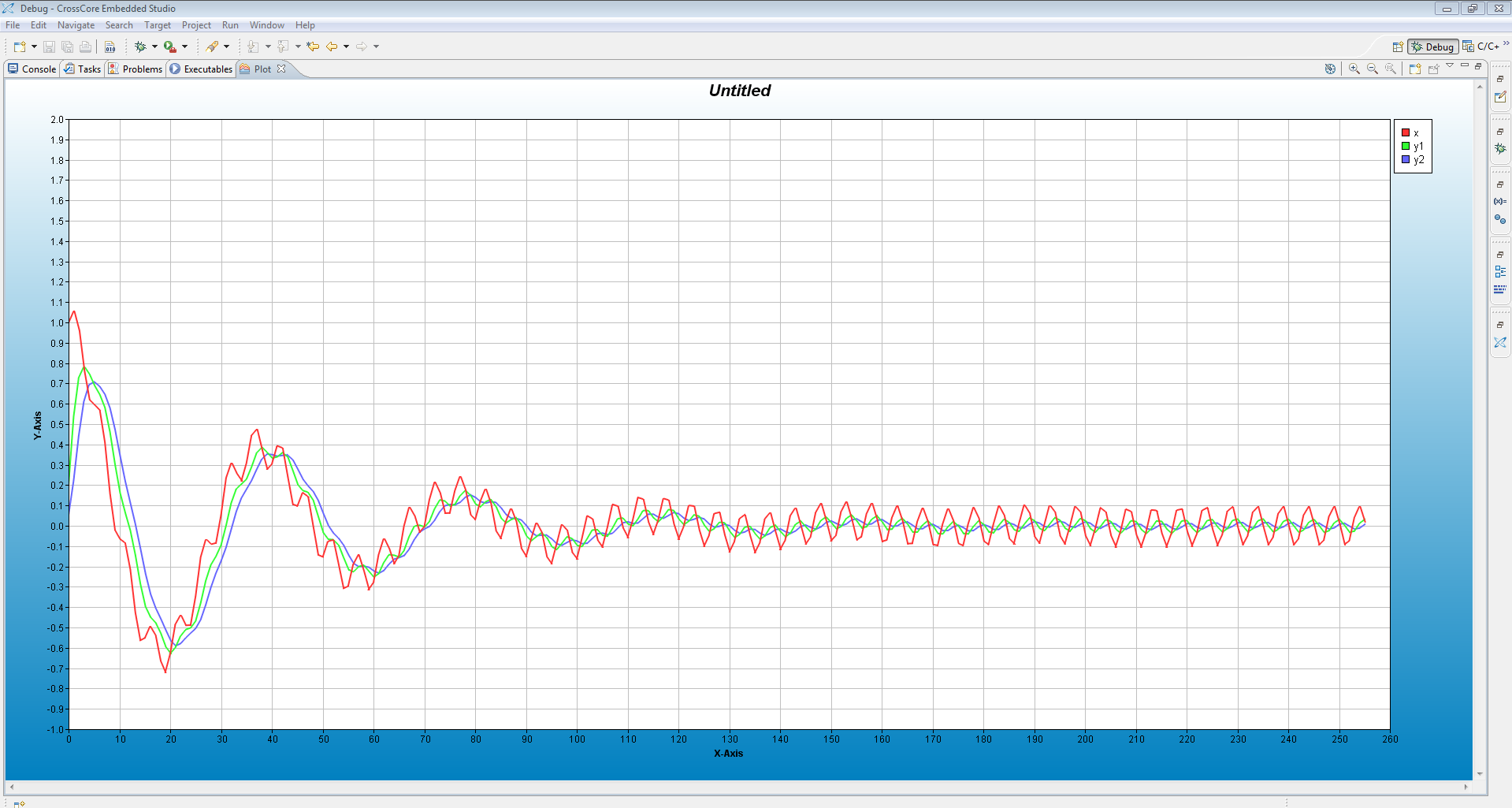
=

As we know:t=1/4.8

X[n]=

After the signal go through two kinds of low pass filters, we get:

Time domain



From the time domain plot, roughly check that the gain of the implemented \_filters agrees

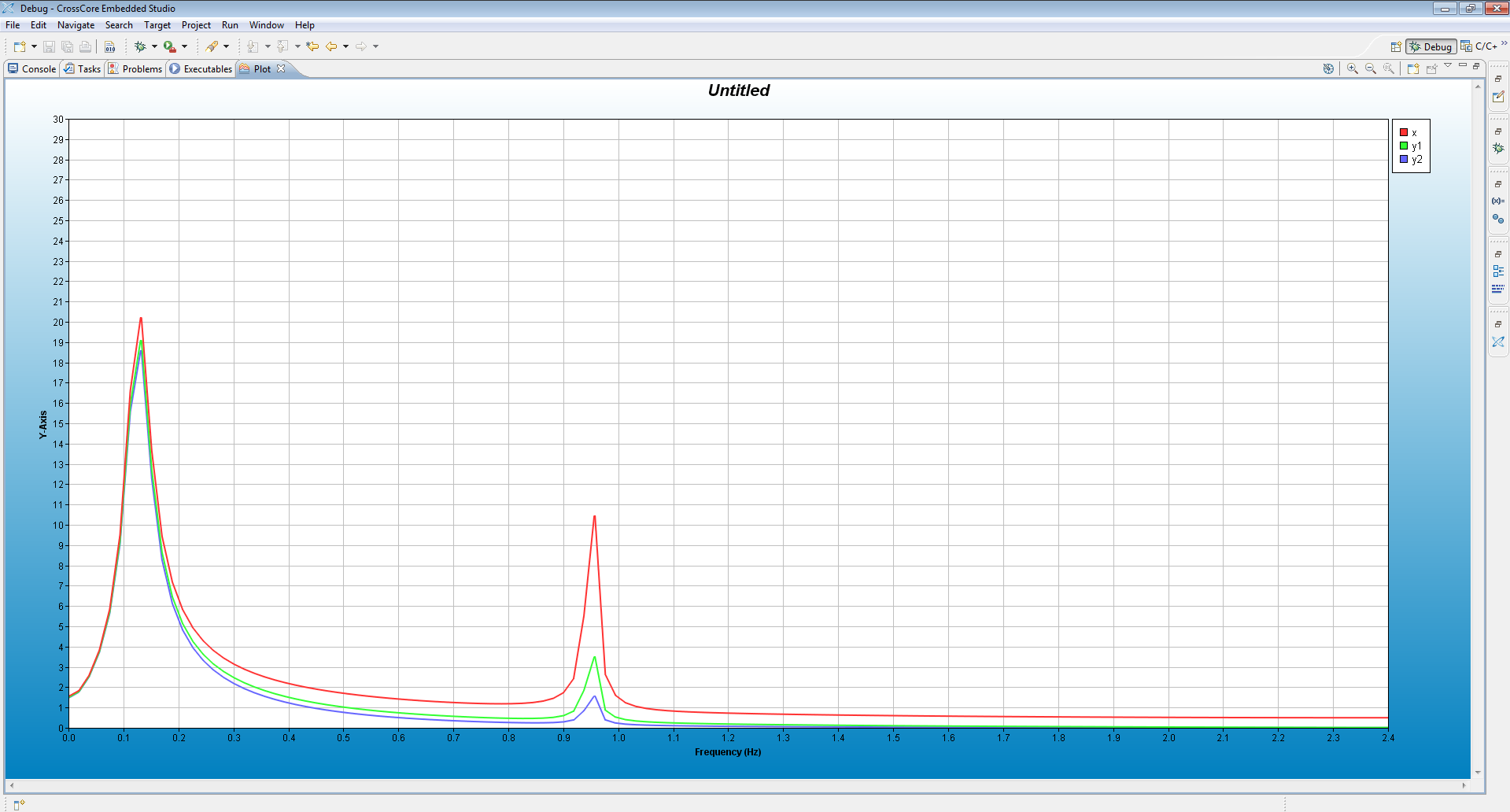
with the gain of the designed \_filters at the frequency of the x2[n] signal.

(这道题要求从时域的图里面分析gain有没有达到设计要求)

In the two images above, x (the red one) means x[n], the sequence without filters, y1 (the green one) means signal go through first order filter, and y2 (the blue one) means signal go through second order filter. From the diagram, we can see at the frequency of 0.95Hz,

the gain=1.7÷10.5=0.162, which is lower than 0.25, thus it suits the filter we designed.

Frequency domain

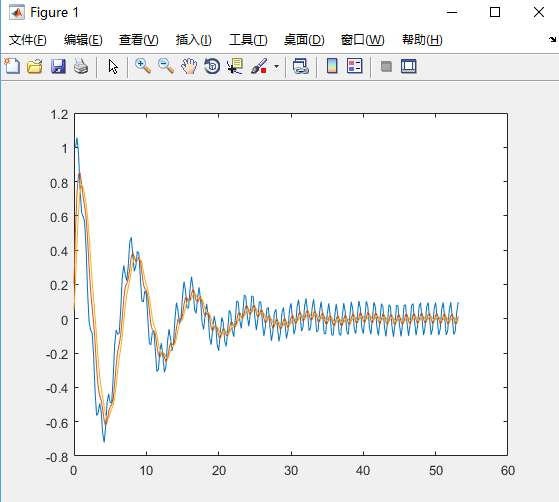
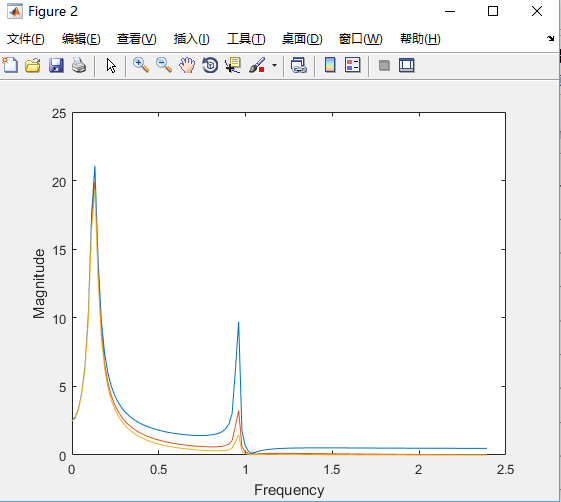


ii)

As seen from the frequency plot, at , the original signal has magnitude of 20, and the signal after the first order signal has magnitude of 19 and that in the second order filter has the same performance, , both filter are satisdying the requirement which retain 95% of the magnitude of the signal.

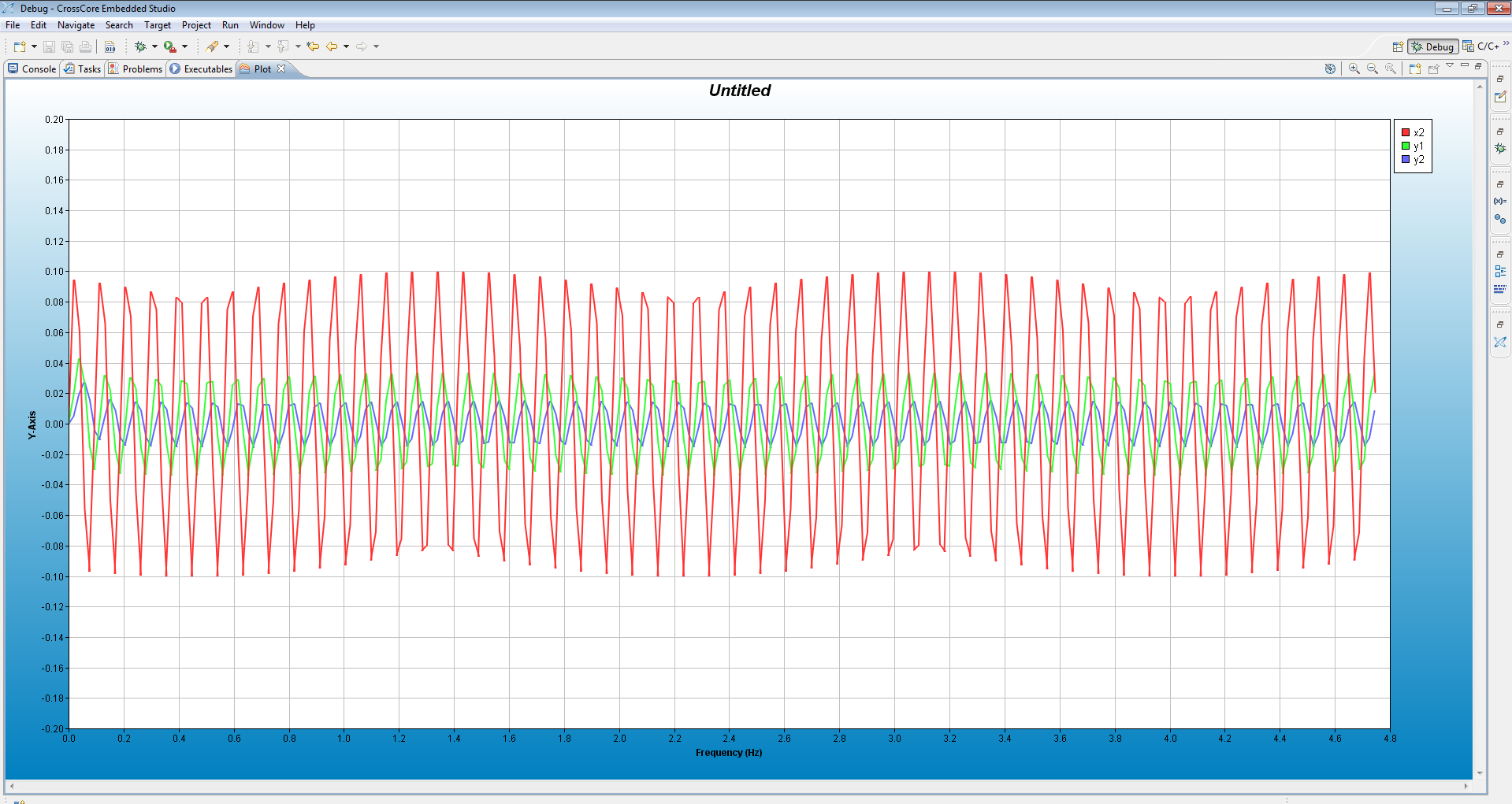
Whereas at , the original signal has magnitude of 10.2, and the signal after the first order signal has magnitude of 3.2, which is , dissatisfying the design requirement, and that in the second order filter is .

We use Matlab to stimulate the signal before implementing on the DSP board. The code and the figures are showed below.

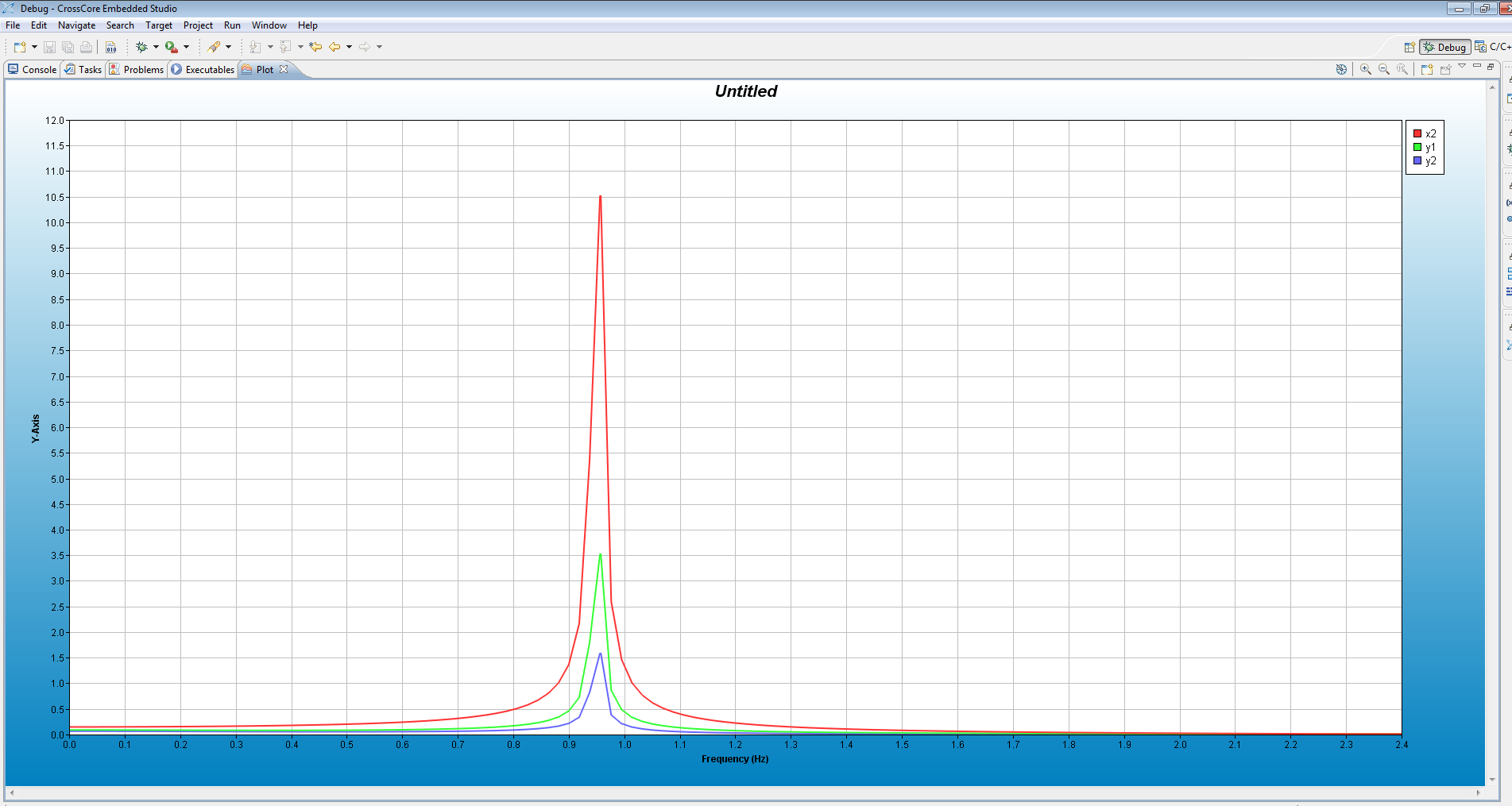
 

Time domain Frequency domain

After signal [n] go through two kinds of filters, we get:



Time domain



Frequency domain

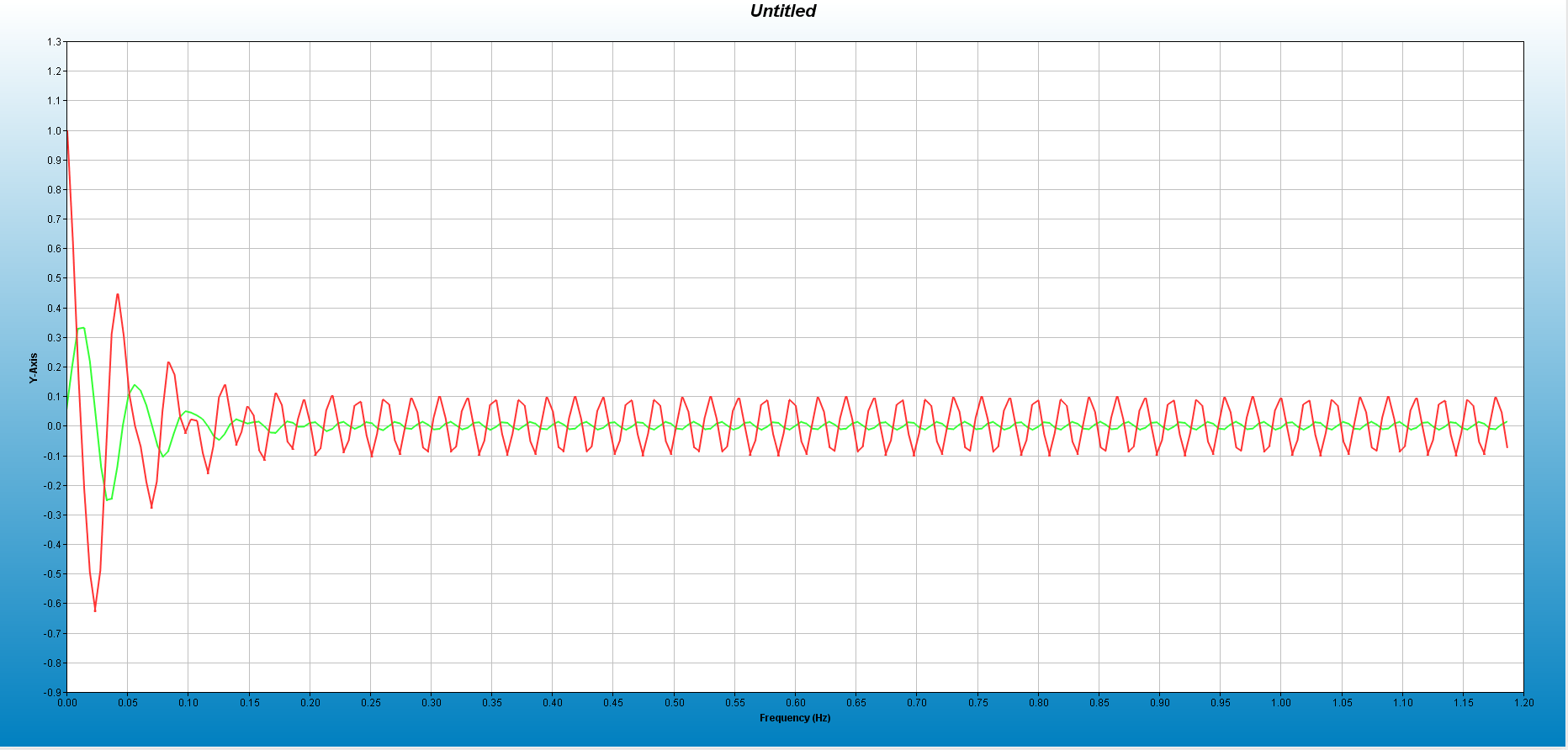
In the two images above, x (the red one) means[n], the sequence without filters, y1 (the green one) means signal go through first order filter, and y2 (the blue one) means signal go through second order filter. From the diagram we can see at the frequency of 0.95Hz,

the gain=after the second order filter, which is lower than 0.25, thus it suits the filter we designed. However the first order filter has percentage gain of. Moreover, After the signal go through the second order filter, it has 90 degrees delay.

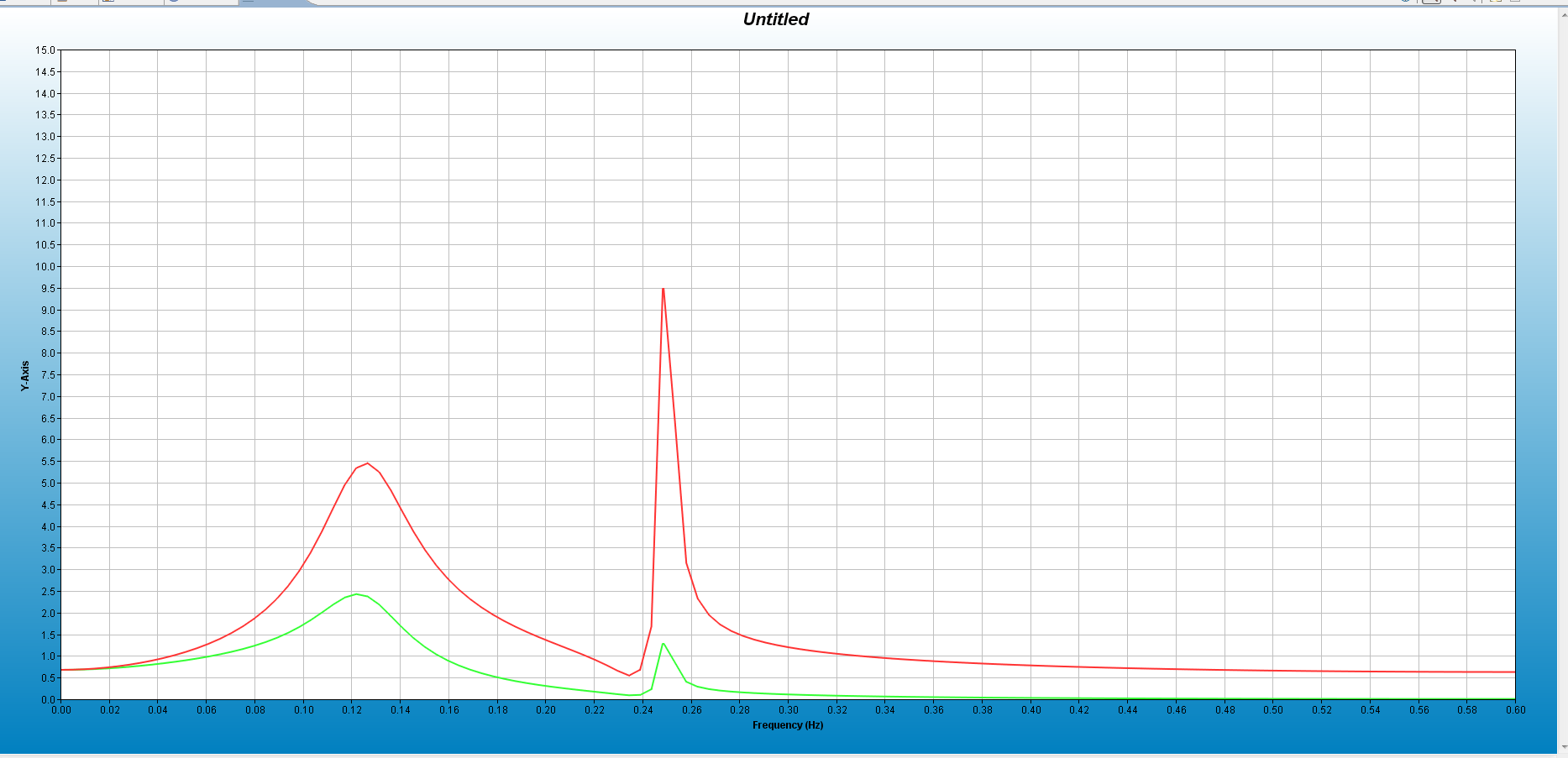
The code is attached below.

|  |
| --- |
|  |
|  | #include <math.h>  #include <stdio.h> |
|  |  |
|  | // Globals |
|  | #define N 256 |
|  | #define PI 3.1415 |
|  |  |
|  | float x[N]; |
|  | float x2[N]; |
|  | float y1[N]; |
|  | float y2[N]; |
|  |  |
|  | int main(void) |
|  | { |
|  |  |
|  | int i; |
|  | float omega1 = 0.25 \* PI, omega2 = 1.9 \* PI; |
|  | float T2 = 1/4.8; |
|  | float T = 1.0; |
|  | float a=0.12; |
|  | float alpha1 = 0.593, alpha2 = 0.464; |
|  |  |
|  |  |
|  | //x[0] = exp(-a\*0\*T2)\*cos(omega1\*0\*T2) + 0.1\*sin(omega2\*0\*T2); |
|  | x2[0] = 0.1\*sin(omega2\*0\*T2); |
|  | y1[0] = ((1-alpha1)/2)\*x2[0]; |
|  | y2[0] = ((1-alpha2)/2)\*y1[0]; |
|  | for (i = 0; i < N; i++) |
|  | { |
|  | //x[i] = exp(-a\*i\*T2)\*cos(omega1\*i\*T2) + 0.1\*sin(omega2\*i\*T2); |
|  | x2[i] = 0.1\*sin(omega2\*i\*T2); |
|  | y1[i] = ((1-alpha1)/2)\*x2[i]+((1-alpha1)/2)\*x2[i-1]+alpha1\*y1[i-1]; |
|  | y2[i] = ((1-alpha2)/2)\*y1[i]+((1-alpha2)/2)\*y1[i-1]+alpha2\*y2[i-1]; |
|  | } |
|  |  |
|  | /\*for (i = 0; i < N; i++) |
|  | { |
|  | printf("x[%d] = %f\n", i, x[i]); |
|  | } |
|  | \*/ |
|  | printf("Done.\n"); |
|  |  |
|  | return 0; |
|  | } |

After the signal go through the second order low pass filter, we get:



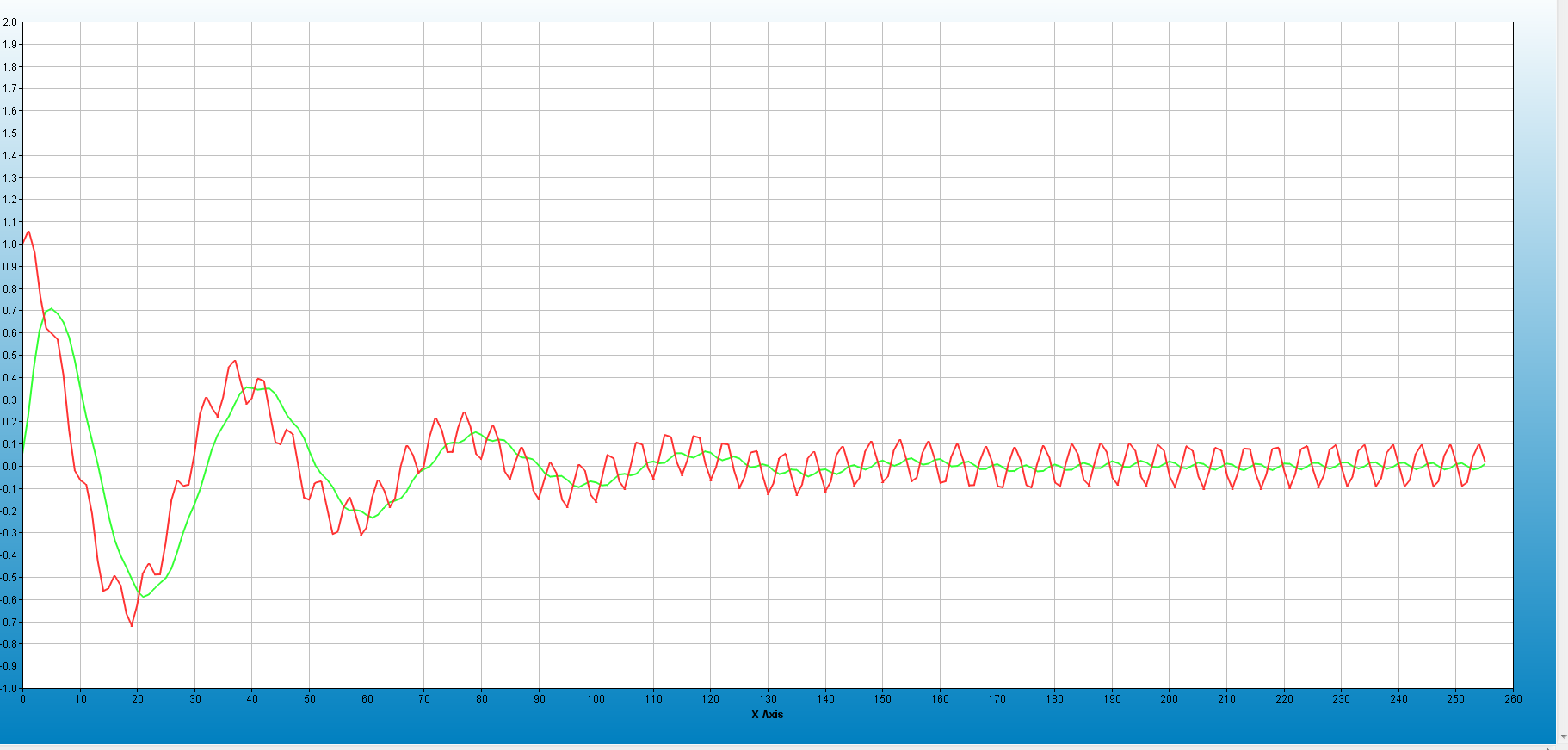
Time domain



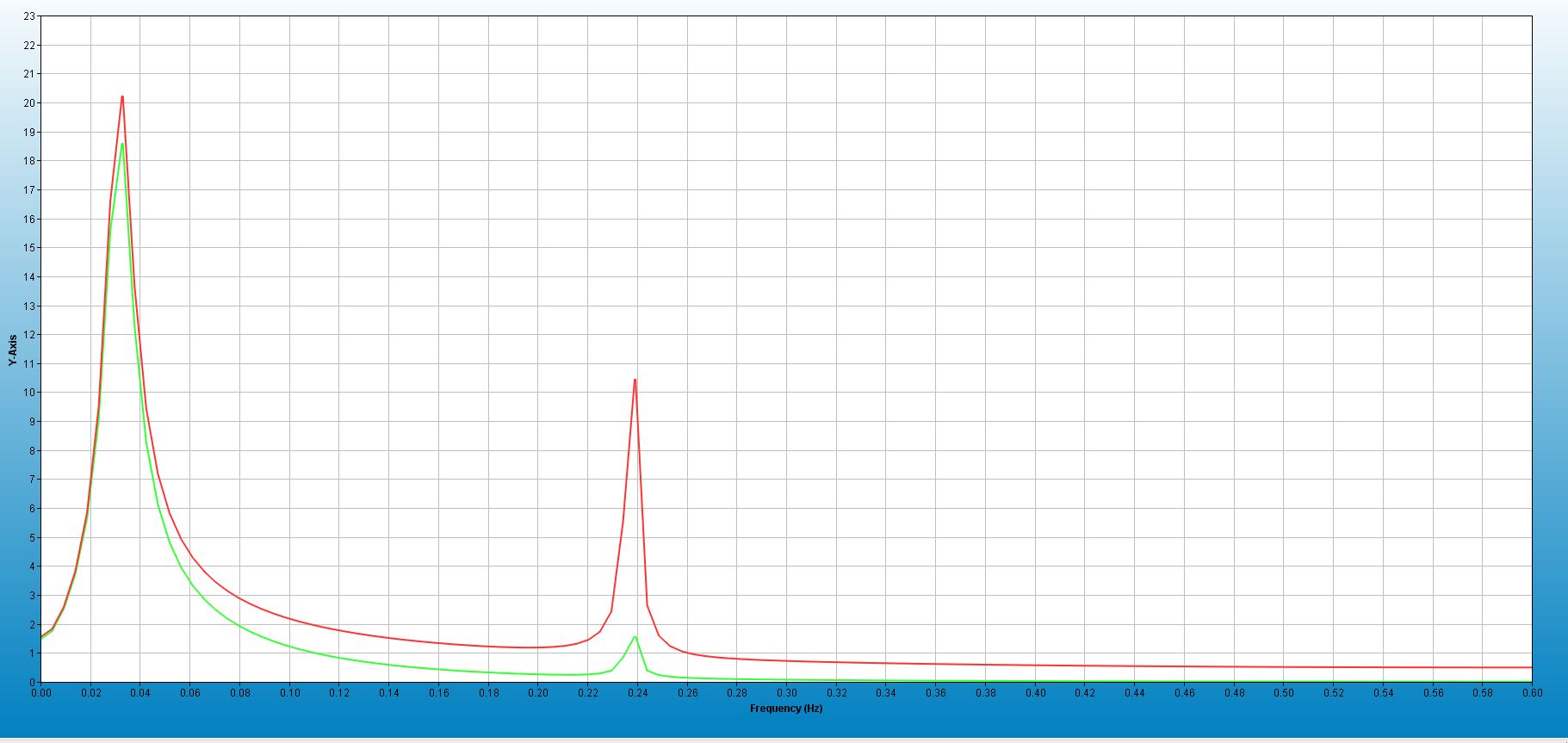
Frequency domain

In the two images above, x (the red one) means x[n], the sequence without filters, y1 (the green one) means signal go through the second order filter.

Then we plot x(t) at 1.2Hz,



Time domain



Frequency domain

Plot this signal in the time and frequency domain and compare it with the signal obtained by sampling x(t) at 1.2 Hz. Comment on the results.

(这道题要求对比 down sample + filter 过之后的信号跟1.2Hz采样率的信号)

After y[n] go through the filter, at about 0.25Hz, the gain =2.3÷19.0=0.121; Then at 0.13Hz,

The gain =5.0÷11.0=0.455. For x(t), when it go through the second order filter, at 0.24Hz,

The gain= 1.5÷10.5=0.143; at 0.03Hz, the gain=18.5÷20.3=0.911. From the analysis we see that the filter act well when signal is x(t), it is a low pass filter.

For the signal is y[n], it is also act as a low pass filter, but it attenuates about a half at 0.13Hz.

The code of how to generate and plot y[n] is attached below.

|  |
| --- |
|  |
|  | #include <math.h>  #include <stdio.h> |
|  | #include <complex.h> |
|  |  |
|  | // Globals |
|  | #define N 256 |
|  | #define PI 3.1415 |
|  |  |
|  | float x[N]; |
|  | float x2[N]; |
|  | float y1[N]; |
|  | float y2[N]; |
|  | float n1[N]; |
|  |  |
|  | int main(void) |
|  | { |
|  |  |
|  | int i; |
|  |  |
|  | float omega1 = 0.25 \* PI, omega2 = 1.9 \* PI; |
|  | float T2 = 1/4.8; |
|  | float T = 1/1.2;; |
|  | float a=0.12; |
|  | float alpha1 = 0.593, alpha2 = 0.464; |
|  |  |
|  |  |
|  | x[0] = exp(-a\*0\*T2)\*cos(omega1\*0\*T2) + 0.1\*sin(omega2\*0\*T2); |
|  | //x2[0] = 0.1\*sin(omega2\*0\*T2); |
|  | y1[0] = ((1-alpha1)/2)\*x2[0]; |
|  | y2[0] = ((1-alpha2)/2)\*y1[0]; |
|  | for (i = 0; i < N; i++) |
|  | { n1[i]=4\*i; |
|  | x[i] = exp(-a\*i\*T2)\*cos(omega1\*i\*T2) + 0.1\*sin(omega2\*i\*T2); |
|  | //x2[i] = 0.1\*sin(omega2\*i\*T2); |
|  | y1[i] = ((1-alpha1)/2)\*x[i]+((1-alpha1)/2)\*x[i-1]+alpha1\*y1[i-1]; |
|  | y2[i] = ((1-alpha2)/2)\*y1[i]+((1-alpha2)/2)\*y1[i-1]+alpha2\*y2[i-1]; |
|  | } |
|  |  |
|  | /\*for (i = 0; i < N; i++) |
|  | { |
|  | printf("x[%d] = %f\n", i, x[i]); |
|  | } |
|  | \*/ |
|  | printf("Done.\n"); |
|  |  |
|  | return 0; |
|  | } |

Appendix:

Matlab code in question e.

Code:

clc

clear all

close all

T1 = 1/1.2;

T2 = 1/4.8;

omega1 = 0.25 \* pi;

omega2 = 1.9 \* pi;

T = 1.0;

i = 0;

a = 0.12;

alpha1 = 0.593;

alpha2 = 0.464;

x(1) = exp(-a\*0\*T2)\*cos(omega1\*0\*T2) + 0.1\*sin(omega2\*0\*T2);

y1(1) = ((1-alpha1)/2)\*x(1);

y2(1) = ((1-alpha2)/2)\*y1(1);

for n = 2:1:256

%only use fs=4.8Hz

x(n) = exp(-a\*i\*T2)\*cos(omega1\*i\*T2) + 0.1\*sin(omega2\*i\*T2);

y1(n) = ((1-alpha1)/2)\*x(n)+((1-alpha1)/2)\*x(n-1)+alpha1\*y1(n-1);

y2(n) = ((1-alpha2)/2)\*y1(n)+((1-alpha2)/2)\*y1(n-1)+alpha2\*y2(n-1);

i = i+1;

end

sample = 0:1:255;

plot(T2\*sample, x);

hold on

plot(T2\*sample, y1);

plot(T2\*sample, y2);

figure;

fs1 = 1.2;

fs2 = 4.8;

N=255;

X\_mags = abs(fft(x))

Y1\_mags = abs(fft(y1));

Y2\_mags = abs(fft(y2));

fax\_bins = [0 : N-1]; %frequency axis in bins

N\_2 = ceil(N/2);

plot(fax\_bins(1:N\_2)\*fs2/N, X\_mags(1:N\_2))

hold on;

plot(fax\_bins(1:N\_2)\*fs2/N, Y1\_mags(1:N\_2))

plot(fax\_bins(1:N\_2)\*fs2/N, Y2\_mags(1:N\_2))

xlabel('Frequency')

ylabel('Magnitude');